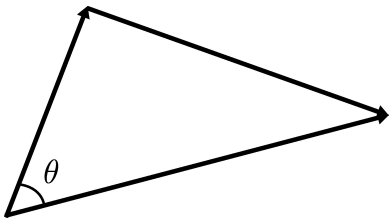
Section 13.3

The Dot Product

Definition: Dot Product
Given component vectors $\vec{u} = \langle u_1, u_2, \ldots, u_n \rangle, \vec{v} = \langle v_1, v_2, \ldots, v_n \rangle$, the **dot product** of $\vec{u}$ and $\vec{v}$ is the scalar

$$\vec{u} \cdot \vec{v} = \sum_{i=1}^{n} u_i v_i$$

The dot product encodes the angle θ between the vectors $\vec{u}$ and $\vec{v}$.



Theorem: Orthogonality
Two vectors $\vec{u}, \vec{v}$ are orthogonal if and only if $\vec{u} \cdot \vec{v} = 0$.

Theorem: Properties of the Dot Product
Suppose $\vec{u}, \vec{v}, \vec{w}$ are vectors and let c be a scalar.

1. $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$
2. $c(\vec{u} \cdot \vec{v}) = (c\vec{u}) \cdot \vec{v} = \vec{u} \cdot (c\vec{v})$
3. $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

Exercise 13.3.1. How much in the direction of $\vec{\imath}$ does the vector $\vec{v} = \langle -1, 2, -1 \rangle$ point?

Exercise 13.3.2. How much in the direction of $\vec{u} = \langle 3, 4 \rangle$ does the vector $\vec{v} = \langle 4, 1 \rangle$ point?

Work Done

If a force given by a vector $\mathbf{F}$ is applied at an angle θ to the direction of motion; the resulting displacement of the object is a vector $\mathbf{d}$. The work done by the force is the component of the force in the direction of the motion (the projection $\mathbf{scal}_d\mathbf{F}$) multiplied by the distance moved by the object $|\mathbf{d}|$. Hence,

$$W = (|\mathbf{F}| \cos\theta)|\mathbf{d}| = \mathbf{F} \cdot \mathbf{d}$$

Example 13.3.3. A force $\mathbf{F} = \langle 3, 3, 2\rangle$ in newtons moves an object along a line segment from $P(1, 1, 0)$ to $Q(6, 6, 0)$ in meters. What is the work done by the force?

Section 13.4

The Cross Product

We want a binary operation on two vectors$\vec{u}, \vec{v}$ that returns a vector that is orthogonal to both $\vec{u}$ and $\vec{v}$. There are infinitely many such vectors so we hope to find some natural choice.

Definition: Cross Product
Let $\vec{u} = u_1\vec{\imath} + u_2\vec{\jmath} + u_3\vec{k}, \vec{v} = v_1\vec{\imath} + v_2\vec{\jmath} + v_2\vec{k}$. The

$$\vec{u} \times \vec{v} = (u_2v_3 - u_3v_2)\vec{\imath} + (u_1v_3 - u_3v_1)\vec{\jmath} + (u_1v_2 - u_2v_1)\vec{k}$$

Theorem: Properties of the Cross Product
Let $\vec{u}, \vec{v}, \vec{w}$ be nonzero vectors in $\mathbb{R}^3$, and let a and b be scalars.

1. $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$
2. $(a\vec{u}) \times (b\vec{v}) = ab(\vec{u} \times \vec{v})$
3. $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w})$
4. $(\vec{u} + \vec{v}) \times \vec{w} = (\vec{u} \times \vec{w}) + (\vec{v} \times \vec{w})$

Magnitude of Cross Product

We have now to find the magnitude and relate this to physical phenomena to classify our definition as "natural."